Vectors

Background Information

Physics and Engineering deal with quantities that have both size and direction. These physical quantities have a special math language in which they are known as vectors. In a science class you may have measured things like volume, mass, length, and area. These quantities all have a size, or magnitude, but do not have a direction. They are known as scalar quantities.

If you have ever given directions you have probably used a type of vector language. "Go 5 blocks North and then turn right and go another 3 blocks." If you just told someone to "walk 5 blocks", without specifying the direction, who knows where they would end up? In physics we use vectors to describe forces, displacement, velocity, and acceleration.

When two or more vectors are added together to get a total vector, this is called the resultant vector. These are all terms that you will be exploring in navigating your Sea Perch.

Introduction to Vectors and Forces

A force is defined as a push or a pull. The S.I. unit for forces are Newtons (N), although in your everyday life you probably use pounds (lb) for force units. The size of the force is important but so is the direction. Imagine a woman is pushing a grocery cart forward with 10 Newtons of force and the force of friction is working against her with a force of 10 N, as illustrated in the picture below.

Force of woman pushing the cart

Force of friction

This could mean a few things. The total force acting on the cart is 0 N because your force is balanced by the force of friction. According to Newton's 2nd Law, if the sum of the forces is zero then the acceleration of the cart is also zero. An acceleration of zero either means the cart is not moving, or it is moving at a constant velocity.

If the woman pushed harder with a 15 N force and the force of friction stayed the same 10 N then the resultant would be 5 N in the direction of the woman's push. It would be like a tug of war between the woman and friction, and she would win! This unbalanced force would cause the wagon to accelerate. What if she pushed the cart with 10 N of force through a sticky puddle on the floor? The stickiness would increase the force of friction to 15 N. In this scenario the friction would win the tug of war and the cart would decelerate. An object

decelerates if the resultant force is in the negative direction. Physically it means the velocity is decreasing over time.

Now think of adding forces in terms of Newton's 2nd Law.

$$\Sigma F = ma$$

For your Sea Perch you will be adding the forces of your motor going forward and the drag of the water going backwards. If the force of the motor is greater than the drag of the water then the resultant force is in the direction of the motor's force and your Sea Perch is accelerating. If the drag is greater than the force of your motor then the resultant force is in the direction of the drag and your Sea Perch will start to slow down. If the forces are exactly equal then the resultant is equal to zero. This is called equilibrium and it means your Sea Perch is either at rest, or moving at a constant velocity.

Using vectors to describe motion

Vectors are also used when describing an object's motion. If your Sea Perch were to be placed in a river that had a southwardly flowing current, then the motion of your Sea Perch would be a combination of the velocity of the river plus the velocity of your Sea Perch. If the water was flowing at 5 m/s, we could represent this with an arrow, a mathematical representation of a vector that is 5 cm long and pointing towards the South.

River Velocity   = 5m/s, South
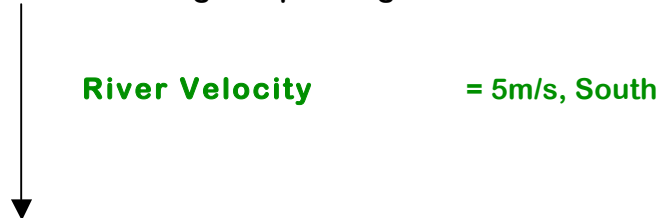
If your Sea Perch was able to move at a maximum speed of 3 m/s and you pointed it due South the vector representing the Sea Perch would like this.

**Sea Perch Velocity**   = 3 m/s, South

What would the total velocity of the Sea Perch relative to the bank?
(Hint: Think about what happens when you are on a moving walkway at the airport.)

Total Velocity   = 8 m/s, South

This picture represents geometric addition of vectors. To add two vectors geometrically, draw the 1st vector. Then draw the 2nd vector so the tail of the 2nd vector touches the head of

the 1st vector. Then draw the resultant vector from the tail of the 1st vector to the head of the 2nd vector. Try to apply this rule to the following sample problems. You need to use a ruler so that your vectors are drawn exactly as the original vectors are drawn.
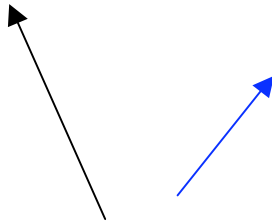
1. (example)
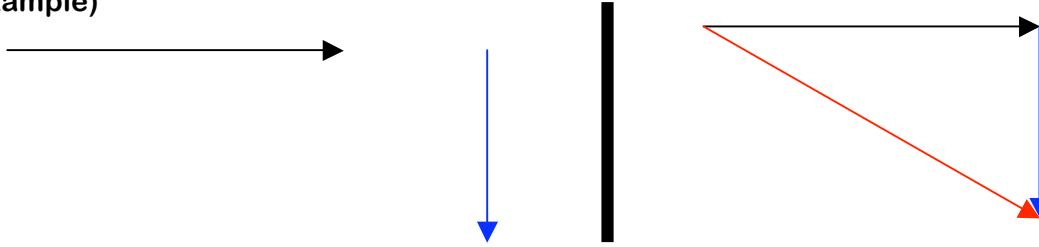
2.

3.

Think of all the different problems we could solve drawing different velocity vectors to find the total velocity of an object. Pilots do this all the time when navigating their airplanes. They have to take into account the velocity of their airplane (both size and direction) in conjunction with the velocity of the wind (both size and direction).

Think of a plane flying West at 10 m/s into a wind that is blowing East at 3 m/s. The following vector diagram could represent this problem.

Plane = 10 m/s, West   Wind = 3 m/s, East

When finding the resultant for vectors pointing in opposite directions think of adding directed numbers. When directed numbers have the same sign you add them together and keep the sign. When directed numbers have opposite signs you take the difference between the numbers and keep the sign of the larger number.

If the plane is flying in the negative direction and the wind is blowing in the positive direction we could write

$$-10 \text{ m/s} + 3 \text{ m/s} = -7 \text{ m/s}$$

The 7 m/s is the size of the resultant velocity vector of the plane and the negative sign shows the direction. This would mean the wind slows down the planes so it is actually moving at 7 m/s, West.

The picture of adding the vectors would look like this.

The red vector represents the resultant of 7 m/s in the Westerly direction.

Use the space below to write your own vector equation using an airplane as an example. Have a peer solve the equation using directed numbers and by drawing the vector picture. Remember your resultant must have both size and direction.

Challenge Question: Now that you have solved these problems geometrically with a ruler, is there a way to find the resultant's size and direction using trigonometry? Discuss this with your teammates.

One way might be to use the Pythagorean theorem.
For example, if a plane if flying east at 30 m/s and a wind is blowing north at 40 m/s, what is the plane's resultant velocity?

Wind = 40 m/s

Resultant = 50 m/s, northeasterly direction

Plane = 30 m/s
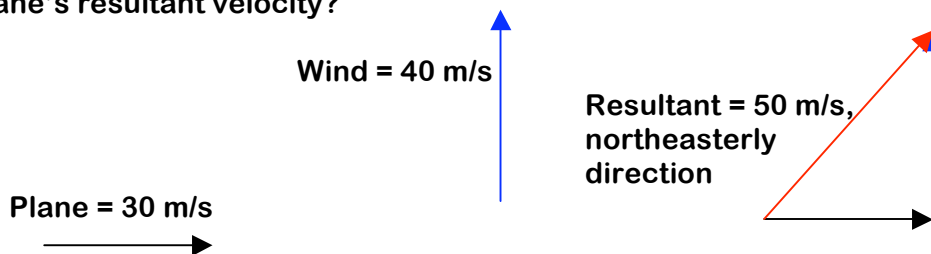
In physics and in calculus the angle is given very specifically using trig to solve for the angle. The positive x-axis, east, is 0^0 and then the angles go counter clockwise around the Cartesian coordinate system. So the positive y- axis, north, is 90^0, the negative x-axis, west is 180^0, and the negative y-axis, south, is 270^0.

So in the example above the direction of the resultant could be found using trigonometry.

50 m/s   40 m/s

θ

30 m/s

$$\text{Sin } \theta = \frac{40\text{m/s}}{50 \text{ m/s}}$$

$$\theta = \text{Sin}^{-1}(0.8)$$

$$\theta = 54^0$$
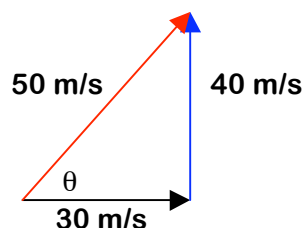
The resultant velocity of the plane would be 50 m/s, at an angle of 54^0.

Is there another way to solve more difficult problems where the vectors are not parallel or perpendicular to each other?

Analytical Addition breaks vectors into component parts so that all the vectors are either parallel or perpendicular to each other. To break a vector into its components sketch the original vector and make it the hypotenuse of a right triangle. Draw the x component first and then the y component. The components are the two vectors that add up to give a resultant equal to the original vector.

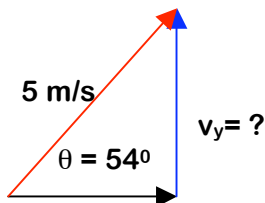If a Sea Perch were moving at 5 m/s, at an angle of 54^0, how much of this velocity would be in the x-direction?

$$\cos 54^0 = v_x / (5\text{m/s})$$

$$v_x = (5\text{m/s}) \text{ x } \cos 54^0$$
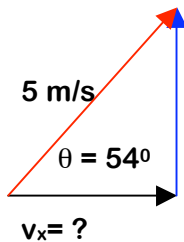
$$v_x = 3 \text{ m/s}$$

How much of this velocity would be in the y-direction?

$$\sin 54^0 = v_y / (5\text{m/s})$$

$$v_y = (5\text{m/s}) \text{ x } \sin 54^0$$

$$v_y = 4 \text{ m/s}$$

How Vectors Relate to your Sea Perch

One of the first challenges you will face is making your Sea Perch neutrally buoyant. Imagine the forces acting on your Sea Perch in the water. Gravity, or the weight of the Sea Perch, will pull down on it. The size of this force will depend on the mass of your Sea Perch. The Buoyant Force will push up on your Sea Perch. This force will depend on how much water your Sea Perch displaces. (Think of when you sit in the tub, you displace a certain amount of water depending on the volume you take up in the tub.)

Physicists draw a free body diagram to show the forces acting on an object.

Buoyant Force

Sea Perch

Weight
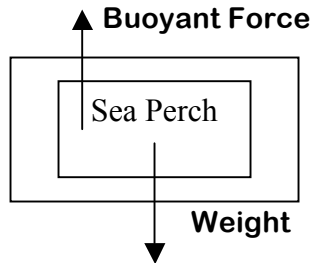
Think about what you have learned about adding vectors together to answer the following questions.

What has to happen to the force vectors if you want your Sea Perch to rise?

How can you accomplish this?

What has to happen to the force vectors if you want your Sea Perch to sink lower in the water?

How can you accomplish this?

One of the other challenges will be to make your Sea Perch move forward using the motors. Each motor produces a force vector that will act on your Sea Perch. To find the resultant motion of your Sea Perch in the horizontal direction you will add the vectors from each motor plus the vector of the water's drag on the Sea Perch.

Draw a free body diagram of the force vectors acting on your Sea Perch in the horizontal direction (forwards and backwards).

What will happen if the forward force of the motors is less than the drag of the water?

How can you use the idea of forces to explain how to move your Sea Perch backwards?

How can you use the idea of vectors to navigate your Sea Perch so that it can turn at a right angle? Think of adding the three force vectors from the motors. Do this either geometrically or analytically.

Additional Resources
- If your school has access to a force table, or if you want to make one this lab can be extended to add more than two vectors. There are numerous force table labs online. Please visit this link for a tutorial that discusses a force table as a part of an exploration of vector addition, http://www.physicsclassroom.com/class/vectors/u3l3a.cfm
- If you would like to learn more about vectors, please visit the following URL: http://www.physicsclassroom.com/class/vectors/u3l3a.cfm
- For some additional information, try the following sites to engage in some interactive simulations http://phet.colorado.edu/en/simulation/vector-addition and http://www.compadre.org/introphys/items/detail.cfm?ID=9403