



## Teacher Content Brief

# Vectors

## Introduction

Your students will need to be able to maneuver their Sea Perch during the competition, so it will be important for them to understand how forces combine to create motion. One way to help your students think about these forces is to consider vectors. This brief will be an introduction to the difference between scalars and vectors. It will also introduce the concepts and math behind vector addition. If you have students with an extensive math background there will also be a small section on the right hand rule and vector multiplication, which is applied in the motor module.

## Standards

Benchmarks 2061 Project (see References section to link to the online standards):

- At the end of 8th grade, students should know that
  - That unbalanced forces acting on an object changes its speed or direction of motion, or both.
  
- At the end of 12th grade, students should know that
  - The change in motion (direction or speed) of an object is proportional to the applied force and inversely proportional to the mass.
  - Whenever one thing exerts a force on another, an equal amount of force is exerted back on it
  - Any object maintains a constant speed and direction of motion unless an unbalanced outside force acts on it.

## Background Information

Physics and Engineering deal with quantities that have both size and direction. These physical quantities have a special math language in which they are known as [vectors](#). In a science class students have measured things like volume, mass, length, and area. These quantities all have a size, or [magnitude](#), but do not have a [direction](#). They are known as [scalar](#) quantities.

Vectors are used to explore forces and motion. In this competition your students will be adding the force vectors acting on their Sea Perch so that it is [neutrally buoyant](#) and so that they can use their [electric motors](#) to navigate it through the prescribed course challenges.

Vectors can be added geometrically and analytically. When two or more vectors are added together the total vector is called the [resultant vector](#). When multiple vectors add up to a resultant of zero, the system is in [equilibrium](#). [Geometric addition](#) uses arrows to represent the size and direction of the vector. These problems can be solved using a ruler and a protractor. Drawing the arrows give the students a mathematical concept of a physical quantity. [Analytic addition](#) would be for more complicated problems and for students with a strong background in trigonometry. This is more useful when vectors are not in the same direction or 180° opposite to that direction. In those specific cases, which are detailed below, students can add vectors like directed numbers. If the vectors cover a variety of directions in all 360° then analytic addition can be used to get very precise answers.

For a much more advanced discussion about vector math students can explore [3-D vectors](#) and [vector multiplication](#). This is not necessary to navigate the Sea Perch but as they continue their physics education they will explore these ideas in depth.

### **Vectors and Forces**

As an introduction talk about different quantities students have measured in science classes. Then have them split this list into two categories, one category **SCALARS**, the quantities have size but no direction. In the other category, **VECTORS**, the quantities have both size and direction. For example,

**SCALARS**  
LENGTH  
DISTANCE  
VOLUME  
MASS  
DENSITY

**VECTORS**  
DISPLACEMENT  
VELOCITY  
ACCELERATION  
WEIGHT / FORCE OF GRAVITY  
BUOYANT FORCE

Circle [forces](#), [velocity](#), and [acceleration](#) as these are the types of vectors your students will be adding. Go over the definitions and make sure students understand and can express why it is important for each of these quantities to include direction.

### **SAMPLE PROBLEM TO GO OVER WITH STUDENTS**

It might help your students to go over this problem using a ruler to draw each scenario into their notes. Have them pick a scale, for example 1 N of force = 1 cm of length in their picture.

A [force](#) is defined as a push or a pull. The S.I. unit for forces are Newtons (N), although in your everyday life you probably use pounds (lb) for force units. The size of the force is important but so is the direction. Imagine a woman is pushing a grocery cart forward with 10 Newtons of force and the force of [friction](#) is working against her with a force of 10 N, as illustrated in the picture below.



This could mean a few things. The total force acting on the cart is 0 N because your force is balanced by the [force of friction](#).

Have the students explain to each other why the resultant is 0 N. They could do this using their picture or adding the vectors like directed numbers.

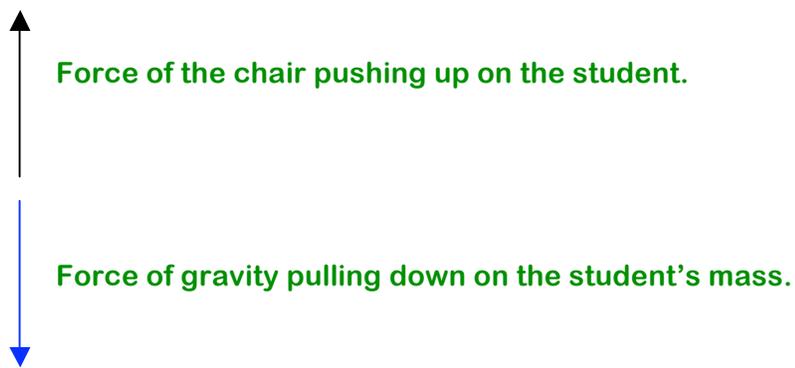
In the picture above the woman's push is in the direction of the positive x-axis so it can be written as +10 N. The force of friction is pointing along the negative x-axis so it could be written as -10 N. The students could then figure out that

$$+10 \text{ N} + (-10 \text{ N}) = 0$$

According to [Newton's 2<sup>nd</sup> Law](#), if the sum of the forces is zero then the [acceleration](#) of the cart is also zero. An acceleration of zero either means the cart is not moving, or it is moving at a constant velocity.

This is an important point to make. Many times students think if the acceleration is zero the object is not moving. Make sure they understand that an acceleration of zero could mean the object is at rest although it could also mean the object is moving at a constant velocity. You could review [Newton's 1<sup>st</sup> Law](#), and point out that an object's motion does not change unless the forces are unbalanced. Or in terms of vectors, the object's motion does not change if the resultant force is zero. If the resultant force on an object is any thing other than zero then the forces are unbalanced and the object's velocity changes.

Another important item to address is that sometimes students think that if an object is not moving there are no forces. For example, while they are sitting in their seats they are not moving. Ask them if there are any forces acting on them. Then depending on their answers, draw a free-body diagram of them sitting on the chairs with two force vectors pushing on them.



What would happen to them if the force of the chair suddenly grew in size? They would momentarily levitate, which would be fun but just doesn't happen.

What would happen if the Force of Gravity suddenly increased in strength? Chances are the chair would break and they would fall down.

Finally, ask the students if all of the chairs are pushing up with the same amount of force. This is a good discussion to have about mass, and how each chair is balancing the force of gravity for each individual student.

After this discussion go back to the woman with the grocery cart. Again, consider having the students solve these problems using free-body diagrams of each force. Have them draw the resultant vector as a dashed line or in a different color.

If the woman pushed harder with a 15 N force and the force of friction stayed the same 10 N then the resultant would be 5 N in the direction of the woman's push. It would be like a tug of war between the woman and friction, and she would win!

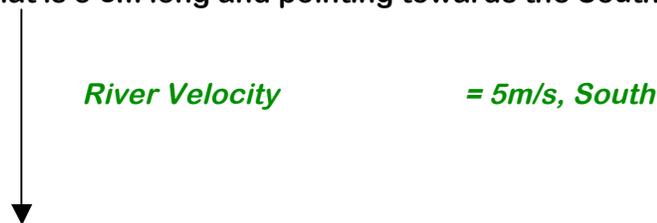
This unbalanced force would cause the wagon to accelerate. What if she pushed the cart with 10 N of force through a sticky puddle on the floor? The stickiness would increase the force of friction to 15 N. In this scenario the friction would win the tug of war and the cart would [decelerate](#).

For your Sea Perch you will be adding the forces of your motor going forward and the drag of the water going backwards. If the force of the motor is greater than the [drag](#) of the water then your Sea Perch is accelerating. If the drag is greater than the force of your motor then your Sea Perch will start to slow down. If they are exactly equal to each other your Sea Perch is either at rest, or moving at a constant velocity.

This might be a good time to draw a free-body diagram of the forces acting on a Sea Perch. The students could draw the horizontal forces, which cause horizontal motion. Then they could draw the vertical forces, which cause vertical motion.

### Using vectors to describe motion

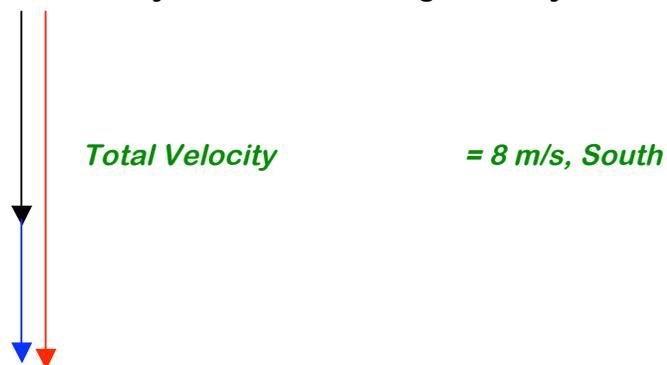
Vectors are also used when describing an object's motion. If your Sea Perch were to be placed in a river that had a southwardly flowing current, then the motion of your Sea Perch would be a combination of the velocity of the river plus the velocity of your Sea Perch. If the water was flowing at 5 m/s, we could represent this with an arrow, a mathematical representation of a vector that is 5 cm long and pointing towards the South.



If your Sea Perch was able to move at a maximum speed of 3 m/s and you pointed it due South the vector representing the Sea Perch would like this.



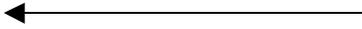
What would the total velocity of the Sea Perch relative to the bank?  
(Hint: Think about what happens when you are on a moving walkway at the airport.)



Think of all the different problems we could solve drawing different velocity vectors to find the total velocity of an object. Pilots do this all the time when navigating their airplanes. They have to take into account the velocity of their airplane (both size and direction) in conjunction with the velocity of the wind (both size and direction).

Think of a plane flying West at 10 m/s into a wind that is blowing East at 3 m/s. The following vector diagram could represent this problem.

Plane = 10 m/s, West



Wind = 3 m/s, East



When finding the resultant for vectors pointing in opposite directions think of adding directed numbers. When directed numbers have the same sign you add them together and keep the sign. When directed numbers have opposite signs you take the difference between the numbers and keep the sign of the larger number.

If the plane is flying in the negative direction and the wind is blowing in the positive direction we could write

$$-10 \text{ m/s} + 3 \text{ m/s} = -7 \text{ m/s}$$

The 7 m/s is the size of the resultant velocity vector of the plane and the negative sign shows the direction. This would mean the wind slows down the planes so it is actually moving at 7 m/s, West.

The picture of adding the vectors would look like this.



The red vector represents the resultant of 7 m/s in the Westerly direction.

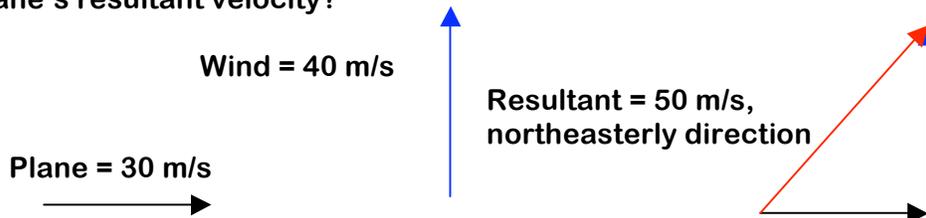
If your students are not sure about the picture, explain that this is geometric addition. The way to find a resultant vector geometrically is to draw the 1<sup>st</sup> vector. Then place the tail of the 2<sup>nd</sup> vector on the head (the arrow point) of the first vector. Then the resultant is drawn from the tail of the 1<sup>st</sup> vector to the head of the 2<sup>nd</sup> vector.

Use the space below to write your own vector equation using an airplane as an example. Have a peer solve the equation using directed numbers and by drawing the vector picture. Remember your resultant must have both size and direction.

### **HIGH SCHOOL LEVEL BACKGROUND INFORMATION**

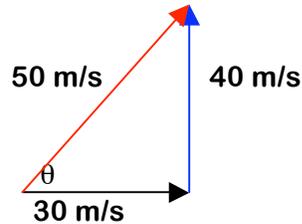
If your students are doing well with these concepts, see if they can think of other vectors they could add that are not along a straight line. If they come up with an example where the vectors are perpendicular to each other then they can use the [Pythagorean Theorem](#) to solve the problem.

For example, if a plane is flying east at 30 m/s and a wind is blowing north at 40 m/s, what is the plane's resultant velocity?



In physics and in calculus the angle is given very specifically using trig to solve for the angle. The positive x-axis, East, is 0° and then the angles go counter clockwise around the Cartesian coordinate system. So the positive y-axis, North, is 90°, the negative x-axis, West is 180°, and the negative y-axis, South, is 270°.

So in the example above the direction of the resultant could be found using trigonometry.



$$\sin \theta = \frac{40\text{m/s}}{50\text{m/s}}$$

$$\theta = \sin^{-1}(0.8)$$

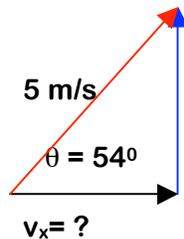
$$\theta = 54^\circ$$

The resultant velocity of the plane would be 50 m/s, at an angle of  $54^\circ$ .

Have students consider problems where the vectors are not in any particular pattern relative to each other. How would these problems be solved? The answer would be analytical vector addition. This will not be explained in depth in this brief.

Any vector can be broken into [component](#) parts. To do this the original vector becomes the hypotenuse of a right triangle. The components are the two vectors that add up to give a resultant equal to the original triangle.

If a Sea Perch was moving at 5 m/s, at an angle of  $54^\circ$ , how much of this velocity would be in the x-direction?

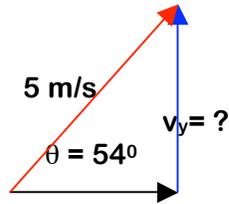


$$\cos 54^\circ = v_x / (5\text{m/s})$$

$$v_x = (5\text{m/s}) \times \cos 54^\circ$$

$$v_x = 3\text{ m/s}$$

How much of this velocity would be in the y-direction?



$$\sin 54^\circ = v_y / (5\text{m/s})$$

$$v_y = (5\text{m/s}) \times \sin 54^\circ$$

$$v_y = 4 \text{ m/s}$$

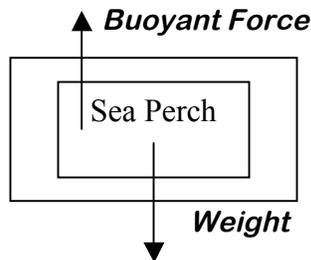
The University of Colorado at Boulder has a website with a lot of helpful simulations. Here is the link to a simulation on vector addition. Students can draw vectors and see the different components. Then they can add vectors using components to find the resultant vector.

<http://phet.colorado.edu/en/simulation/vector-addition>

### **How Vectors Relate to your Sea Perch**

One of the first challenges you will face is making your Sea Perch neutrally buoyant. Imagine the forces acting on your Sea Perch in the water. Gravity, or the weight of the Sea Perch, will pull down on it. The size of this force will depend on the mass of your Sea Perch. The Buoyant Force will push up on your Sea Perch. This force will depend on how much water your Sea Perch displaces. (Think of when you sit in the tub, you displace a certain amount of water depending on the volume you take up in the tub.)

Physicists draw a free body diagram to show the forces acting on an object.



Think about what you have learned about adding vectors together to answer the following questions.

What has to happen to the force vectors if you want your Sea Perch to rise?

How can you accomplish this?

What has to happen to the force vectors if you want your Sea Perch to sink lower in the water?

How can you accomplish this?

One of the other challenges will be to make your Sea Perch move forward using the motors. Each motor produces a force vector that will act on your Sea Perch. To find the resultant motion of your Sea Perch in the horizontal direction you will add the vectors from each motor plus the vector of the water's drag on the Sea Perch.

Draw a free body diagram of the force vectors acting on your Sea Perch in the horizontal direction (forwards and backwards).

What will happen if the forward force of the motors is less than the drag of the water?

How can you use the idea of forces to explain how to move your Sea Perch backwards?

### **Additional Resources**

If your school has access to a force table, or if you want to make one this lab can be extended to add more than two vectors. There are numerous force table labs online. Please visit this link for a tutorial that discusses a force table as a part of an exploration of vector addition, <http://www.physicsclassroom.com/class/vectors/u3l3a.cfm>

### **Additional Resources to Share with Students**

- If you would like to learn more about vectors, please visit the following URL: <http://www.physicsclassroom.com/class/vectors/u3l3a.cfm>
- For some additional information, try the following sites to engage in some interactive simulations <http://phet.colorado.edu/en/simulation/vector-addition> and <http://www.compadre.org/introphys/items/detail.cfm?ID=9403>

## **GLOSSARY**

**Vectors**: a quantity with both size and direction

**Magnitude**: the size of a quantity

**Scalar**: a quantity with size only

**Neutrally buoyant**: An object that neither rises nor sinks due to the balance between its weight and the weight of the water it has displaced.

**Electric motors**: A motor that converts electrical energy into mechanical energy.

**Resultant vector**: The sum of two or more vectors.

**Equilibrium**: When the resultant vector adds up to zero.

**Geometric addition of vectors** : Addition of two vectors by drawing arrows to represent physical quantities. The scale of the picture represents the magnitude and direction of the vector. Draw the 1<sup>st</sup> vector. Place the tail of the 2<sup>nd</sup> vector on the head of the first vector. Draw the resultant vector from the tail of the first vector to the head of the 2<sup>nd</sup> vector.

**Analytic addition**: Addition of two vectors using trigonometric functions and the Pythagorean theorem. Vectors are broken into components and then the components are added together.

**3-D vectors**: A vector that has three components, in the plane of the x-axis, y-axis, and z-axis.

**vector multiplication**: Vectors are multiplied using the dot product or the cross product. The dot product, also known as the scalar product, takes two vectors and multiplies them and produces a scalar. The cross product, also known as the vector product, takes two vectors and multiplies them producing a new vector quantity.

**Forces**: A push or pull.

**Velocity**: The ratio of an object's displacement over time.

**Weight/Force of Gravity**: The force of attraction between two objects with mass.

**Buoyant Force**: The upward pressure exerted upon a floating body by a fluid, which is equal to the weight of the body.

**Force of Friction**: The force that opposes motion.

**Force of Drag**: The force that opposes motion through a fluid.

**Acceleration**: The change in velocity over the change in time.

**Displacement**: The distance from the starting point to the end point in a straight line.

**Deceleration**: A negative acceleration, or when an object is slowing down.

**Newton's 1<sup>st</sup> Law**: An object at rest or in constant motion remains at rest or in constant motion until an unbalanced force is applied.

**Newton's 2<sup>nd</sup> Law**: The sum of the forces equals mass times acceleration.

**Vector Components**: Two vectors that add up to form an original vector.

**Pythagorean Theorem**: A relationship between the sides of a right triangle.

$$A^2 + B^2 = C^2$$